December 16, 2021, 14:30 -16:00, online via Zoom

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What Lie algebras can tell us about Jordan algebras

Abstract:

This talk is based on joint work with Florio Ciaglia and Jürgen Jost.

We develop a structure theory for finite dimensional Jordan algebras by defining a generalized distribution on its dual $\mathcal{J}^\star$, canonically determined by the Jordan bracket and invariant under the action of what we call the *structure group of* $\mathcal{J}$. We show that at regular points of $\mathcal{J}^\star$, this distribution carries a naturally defined pseudo-Riemannian metric $\mathcal{G}$.

For *positive* Jordan algebras, we classify the orbits of $\mathcal{J}^\star$ under the structure group action and show that the only regular orbits are those in the closure of the cone of squares or its negative, and these are the only orbits where $\mathcal{G}$ is (positive or negative) definite.

We discuss applications of our construction to both classical and quantum information geometry by showing that, for appropriate choices of $\mathcal{J}$, we may interpret $\mathcal{G}$ as the Fisher-Rao-metric or the Bures-Helstrom metric for a finite sample space.