The difference body as prototype of a valuation under volume constraints

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Let K be a convex body (non-empty compact and convex set) in $\mathbb{R}^n$ and let $\mathcal{K}^n$ denote the family of convex bodies. The difference body of K is the convex body defined as

$$K - K := \{x - y : x, y \in K\}.$$

In the literature there are several characterizations of the difference body. As an operator it is known to be the only translation invariant, continuous and $SL(n)$-covariant Minkowski valuation, as well as the only o-symmetrization which is continuous $GL(n)$-covariant and translation invariant. None of these characterizations makes use of the celebrated Rogers-Shephard inequality, which provides sharp upper bounds for the volume of the difference body, in terms of the volume of the preimage.

After reviewing the role of a Rogers-Shephard type inequality in classifying the difference body, in particular, that the difference body is the only continuous and $GL(n)$-covariant o-symmetrization which enjoys a Rogers-Shephard type inequality, we will focus on Minkowski valuations satisfying certain volume constraints inspired by the Rogers-Shephard inequality.

If ϕ is a Minkowski valuation, we say that ϕ satisfies a *volume constraint* if there exist constants $c, C > 0$ such that for any convex body K

$$c \operatorname{vol}(K) \leq \operatorname{vol}(\phi(\mathrm{K})) \leq C \operatorname{vol}(K).$$

We will prove that only two families of Minkowski valuations satisfy this condition (together with continuity and translation invariance). The first family of operators has as images only cylinders over $(n-1)$-dimensional convex bodies while the second one is essentially made of 1-homogeneous operators.

This classification, in turn, provides us with new classification results for the difference body operator.